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APPLICATION OF THE COORDINATEWISE DESCENT METHOD ON A UNIT INTERVAL FOR WEIGHT OPTIMIZATION OF STRUCTURES MADE OF COMPOSITE MATERIALS

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Problems of weight optimization of anisotropic structures with restrictions on strength, stability, etc., are solved by the coordinatewise descent method on a unit interval, combined with effective methods of nonlinear programming (dichotomy and golden section methods).

Key words: *weight optimization, coordinatewise descent, nonlinear programming, method of boundary integral equations, composite materials, panel with a hole, combined loads.*

Introduction. The necessity of developing methods of rational design of structures made of layered composite materials is caused by permanent extension of their applications in various fields of engineering, in particular, in aircraft building. There are numerous publications dealing with investigations of various elements of such structures [1, 2]. Implementation of scientific research results on composite structures, however, is too slow for the needs of designers. The reasons are the complicated structure of composites, specific features of their deformation properties, instability of their characteristics, and sensitivity to stress concentrators.

The present paper describes the solution of weight optimization problems for anisotropic structures with restrictions on strength, stability, etc., by the coordinatewise descent method on a unit interval. The stress–strain state of plates is determined by the mechanisms of the boundary integral equations [3], which are highly efficient for problems with large gradients of stresses caused by the presence of holes and other stress concentrators. Examples of optimization are a cantilever beam under bending, a flat panel loaded in its plane, and a panel weakened by an elliptical hole and loaded by bending and rotational moments.

1. Method of Coordinatewise Descent on a Unit Interval. Structural design has to satisfy a considerable number of contradictory requirements, for instance, the minimum weight under certain restrictions on strength, stability, lifetime, etc. These problems are solved by optimal design methods (gradient methods, random search method, etc. [4, 5]), where the rigidity parameters of elements are varied in a manner that ensures the minimum weight under imposed restrictions. The parameters are varied in the interval $[0;\infty)$. In the present paper, we achieve this goal by using the coordinatewise descent method on a unit interval, which was proposed in [6–8]. This procedure allows us to use optimization methods (dichotomy and golden section methods) on a finite interval [9].

Let us consider a structural element in the form of a panel (a packet of n layers of different materials) loaded by a spectrum of loads N^p (N is the load vector and $p = 1, \ldots, P$ is the ordinal number of the loading variant). Each layer has the following properties; thickness h_k, specific weight γ_k, and specified elastic, strength, and geometric characteristics (e.g., the angle of packing φ_k). A certain set of restrictions on strength, stability, etc., should be satisfied for each layer $k = 1, \ldots, n$ and each loading variant p: $(\Phi_k^i)_p(h_1, \ldots, h_n) \leqslant 1$ ($i = 1, \ldots, I$, where I is the

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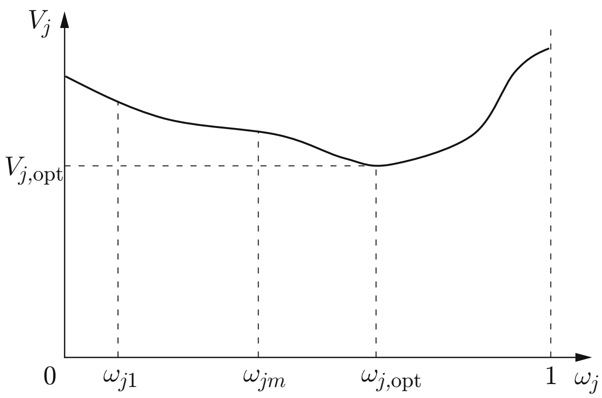

Fig. 1

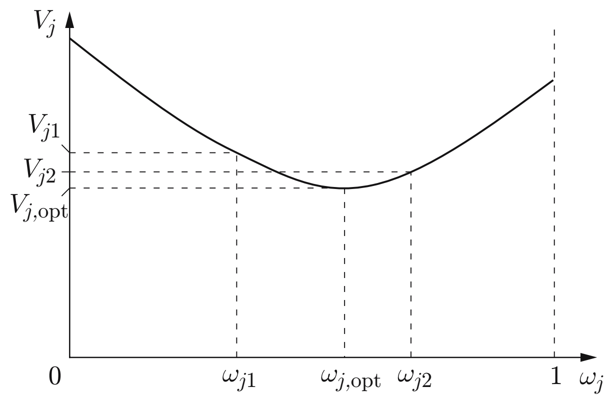

Fig. 2

Fig. 1. Illustration to the coordinatewise descent method.

Fig. 2. Illustration to the optimization algorithm based on the golden section method.

number of restrictions taken into account). The panel weight is proportional to $V = \sum_{k=1}^{n} \gamma_k h_k$. The equation of the envelope of the restriction functions has the form

$$\Phi(h_1, \ldots, h_n) = \max_{i,k,p} \Big\{ (\Phi_k^i)_p (h_1, \ldots, h_n) \Big\} = 1.$$

The optimization problem consists in finding the value of $\min\limits_{h_1,\ldots,h_n} V$ under specified loads N^p $(p = 1, \ldots, P)$ and restrictions $\Phi(h_1, \ldots, h_n) = 1$, i.e., it is a multiparametric problem.

Let us introduce new optimization parameters whose range of variation is a finite interval. The relative weight of the layer is $\omega_k = h_k \gamma_k / V$ $(k = 1, \ldots, n)$. The values of ω_k belong to the segment $[0; 1]$. Let us fix an arbitrary number j such that $1 \leqslant j \leqslant n$. Let $\omega^* = \max\limits_{k \neq j} (\omega_k)$. We introduce relations between the weights of the layers as $\bar{\omega}_k = \omega_k / \omega^*$, $k = 1, \ldots, j-1, j+1, \ldots, n$, $\bar{\omega}_k \in (0; 1]$. As was demonstrated in [7], the thickness of each layer is uniquely determined for fixed values of V, j, ω_j, and $\bar{\omega}_k$ $(k \neq j)$.

The weight of the structure is minimized consecutively. At the first stage, we assume that $j = 1$. We fix the relation between the thicknesses of the remaining layers $\bar{\omega}_k = 1$ $(k \neq 1)$ and then construct the dependence $V_j = V_j(\omega_j)$ by checking consecutively the values of ω_j (Fig. 1). The minimum value of $V_j = V_{j,\mathrm{opt}}$ determines the value of $\omega_{j,\mathrm{opt}}$. At the second stage, we assume that $j = 2$, etc.

Let us consider the situation where the dependence $V_j = V_j(\omega_j)$ is a concave function [7]. This allows us to use the golden section method [9], which leads to a substantial speedup of the optimization process (Fig. 2). In Fig. 2, $\omega_{j1} = 0.38$ and $\omega_{j2} = 0.62$.

2. Formulation of the Problem of Weight Optimization of Structures Made of Composite Materials. A composite structure is a packet of layers $k = 1, \ldots, n$ made of different materials (for instance, those reinforced by unidirectional fibers) and subjected to the action of P different variants of static loads. Let us determine the minimum weight of the composite structure

$$V = \sum_{k=1}^{n} h_k \gamma_k \to \min, \qquad h = \sum_{k=1}^{n} h_k$$

under conditions of strength provision simultaneously for all P variants of static loading and specified fracture criteria for each layer. The varied parameters here are the layer thicknesses h_k. The critical load for the structure as a whole is assumed to be the load at which the weakest layer fails.

In solving the optimization problem, we use the deformation criterion of strength proposed by Tsai and Wu [10], which can be written in terms of strains for the kth layer as follows:

$$\Phi_p^k = G_{ij}^k \varepsilon_i^p \varepsilon_j^p + G_i^k \varepsilon_i^p \leqslant 1, \qquad k = 1, \ldots, n, \quad p = 1, \ldots, P, \quad i, j = 1, 2, 3.$$

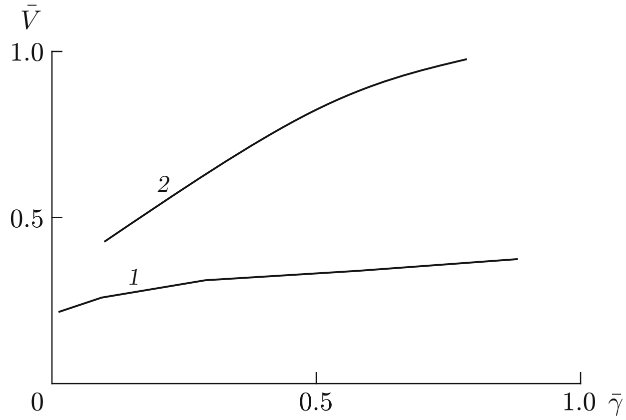

Fig. 3. Weight of the optimal structure versus the relative specific weight of the filler: layered beam model (1) and Timoshenko beam model (2).

Here $\varepsilon_1^p = \varepsilon_x^p$, $\varepsilon_2^p = \varepsilon_y^p$, and $\varepsilon_3^p = \gamma_{xy}^p$ are the strains of the packet in the pth variant of loading. The constants G_{ij}^k and G_i^k are calculated via the elastic and strength characteristics of the material of the kth layer.

Let us introduce the notation $\Phi = \max\limits_{k,p}\{\Phi_p^k\}$ and formulate the optimization problem for a multilayer panel as follows: for specified static loads, elastic characteristics of materials, and angles of packing of the layers, we have to find the layer thicknesses h_k such that they ensure the minimum weight of the plate

$$\min V = \min \sum_{k=1}^{n} V_k = \min \sum_{k=1}^{n} \gamma_k h_k$$

under the constraint $\Phi(h_1, \ldots, h_n) = 1$. The constraint Φ is satisfied and the optimal values of ω_j are determined within 0.05%. Optimization is terminated as soon as the change in the panel weight becomes smaller than 0.01% with respect to the previous iteration.

3. Optimal Design of a Cantilever Beam. A multilayer cantilever beam is loaded by a system of independent loads: distributed sinusoidal load L_0, uniformly distributed load L_1, concentrated force L_2, and moment L_3 on the free edge. The external load-bearing layers are made of composite materials, and the layer in the middle is a light filler.

The parameters known for the beam are the elastic characteristics of the orthotropic composite material (Young's moduli E_1, E_2, G_{12}, and Poisson's ratios ν_1 and ν_2), strength limits under tension and compression in the reinforcement direction X_t and X_c and in the transverse direction Y_t and Y_c, and shear strength limit in the plane of the layer S for the unidirectionally reinforced layer.

We assume that the beam may be subjected to P different variants of static loads: $L^p = (L_0^p, L_1^p, L_2^p, L_3^p)$ ($p = 1, \ldots, P$). The basic relations and the method of calculating the stress–strain state of this structure can be found in [11–13].

Results of optimization for a cantilever beam of length $l = 10$ m loaded on the free end by a concentrated force $L_2 = 10^6$ N are presented below. The beam consists of three layers. The outer layers are made of the HMS/3002M material with the following elastic and strength characteristics: $E_1 = 185$ GPa, $E_2 = 6.76$ GPa, $G_{12} = 5.86$ GPa, $\nu_1 = 0.2$, $X_t = 680$ MPa, $X_c = 690$ MPa, $Y_t = 16$ MPa, $Y_c = 186$ MPa, and $S = 72$ MPa. The fibers are aligned with the longitudinal axis of the beam. The intermediate layer is made of a light filler with small elastic characteristics: $E_1 = E_2 = 0.76$ GPa, $G_{12} = 0.86$ GPa, $\nu_1 = 0.2$, $X_t = X_c = Y_t = Y_c = 7$ MPa, and $S = 2$ MPa. Let us study the effect of the relative specific weight of the filler $\bar{\gamma}$, which is equal to the ratio of the specific weight of the filler to the specific weight of the load-bearing layers, on the weight of the optimal structure (curve 1 in Fig. 3). Results of optimization obtained by the Timoshenko beam theory are also plotted for comparison (curve 2). In this case, the optimal weight is determined by the formula

$$\bar{V} = 2\sqrt{\bar{\gamma}}\,\sqrt{L_2 l (X_t + X_c)/(X_t X_c)}.$$

TABLE 1

Optimal Thicknesses of the Panel Made of the AS4/3502 Material
Obtained by the Coordinatewise Descent Method

Type of loading	h, mm	h_k, mm			
		$\varphi_1 = 45°$	$\varphi_2 = -45°$	$\varphi_3 = 90°$	$\varphi_4 = 0°$
1	10.125	2.375	0	4.625	3.125
2	8.000	3.500	2	2.500	0

TABLE 2

Optimal Thicknesses of the Panel Made of the AS4/3502 Material
Obtained by the Gradient Descent Method

Type of loading	h, mm	h_k, mm			
		$\varphi_1 = 45°$	$\varphi_2 = -45°$	$\varphi_3 = 90°$	$\varphi_4 = 0°$
1	10.10	2.54	0	4.71	2.85
2	7.65	3.75	2.03	1.87	0

4. Optimal Design of Flat Composite Panels Loaded in Their Plane. A flat composite panel with a symmetric structure with respect to the mid-plane consists of n layers of different materials. This panel is in a stress–strain state induced by the action of P different variants of loading. The relation between the forces and strains is determined by the formulas [14]

$$N^p = A\varepsilon^p, \qquad p = 1, \ldots, P,$$

where $N^p = (N_x, N_y, N_{xy})_p^{\mathrm{t}}$ is the vector of forces acting on the packet, $\varepsilon^p = (\varepsilon_1, \varepsilon_2, \varepsilon_{12})_p^{\mathrm{t}}$ is the vector of strains of the packet, and A is the square matrix of coefficients, which depends on the thicknesses h_k and Young's moduli of all materials.

The constraint Φ is satisfied and the optimal values of ω_j are determined within 0.1%. Optimization is terminated as soon as the change in the panel weight becomes smaller than 1% with respect to the previous iteration.

Results presented below were obtained by the optimization procedure for a composite panel made of the AS4/3502 material with the following elastic and strength characteristics: $E_1 = 139.97$ GPa, $E_2 = 10.27$ GPa, $G = 5.72$ GPa, $\nu_{12} = 0.3$, $X_t = 1862$ MPa, $X_c = 1482$ MPa, $Y_t = 52$ MPa, $Y_c = 206$ MPa, and $S = 81$ MPa. The panel consists of the layers (the layer is understood as the set of all monolayers with an identical orientation) with the packing angles $\varphi_1 = 45°$, $\varphi_2 = -45°$, $\varphi_3 = 90°$, and $\varphi_4 = 0°$. Two variants of loading with different values of the load-vector components are considered:

1) $\bar{N}^1 = (3$ MN/m, 1 MN/m, and 1 MN/m$)$ and $\bar{N}^2 = (0$, 4 MN/m, and 0.5 MN/m$)$;

2) $\bar{N}^1 = (1$ MN/m, 3 MN/m, and 0$)$ and $\bar{N}^2 = (0$, 1 MN/m, and 2 MN/m$)$.

Results of optimization are summarized in Table 1. It is seen that loading of the first type leads to degeneration of the layer with the packing angle $-45°$, and loading of the second type leads to degeneration of the layer with the packing angle 0°. For comparison, Table 2 gives the results of solving the same problem by the gradient descent method [14].

The good agreement of the total thicknesses of the packet shows that both methods allow us to reach the neighborhood of the point with the minimum weight of the structure. Some difference in the distributions of thicknesses of individual layers can be attributed to a rather small slope of the objective function.

5. Optimal Design of Composite Panels Weakened by an Elliptical Hole with Bending under the Action of Moments. We consider a plate with a symmetric structure, where the layers with identical characteristics are located symmetrically with respect to the mid-plane (Fig. 4). In plates with symmetric alignment of the layers, each pair of layers (composed of identical layers located above and below the mid-plate at an identical distance from the latter) can have an arbitrary orientation (Fig. 5).

In practice, reinforced layers in all known layered materials reinforced both by fabric and by arbitrarily oriented fibers are orthotropic. Relations that allow obtaining flexural rigidities of the layered material are known for this case [15]. The stress–strain state of the multilayer panel is determined on the basis of the classical Kirchhoff's

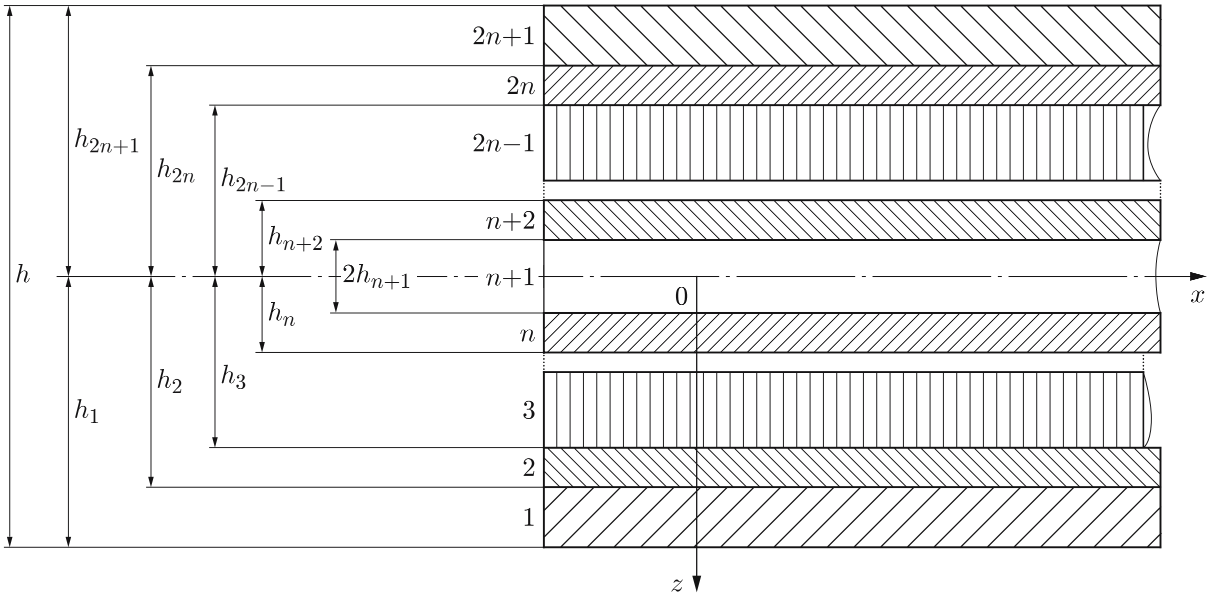

Fig. 4. Structure of the composite panel.

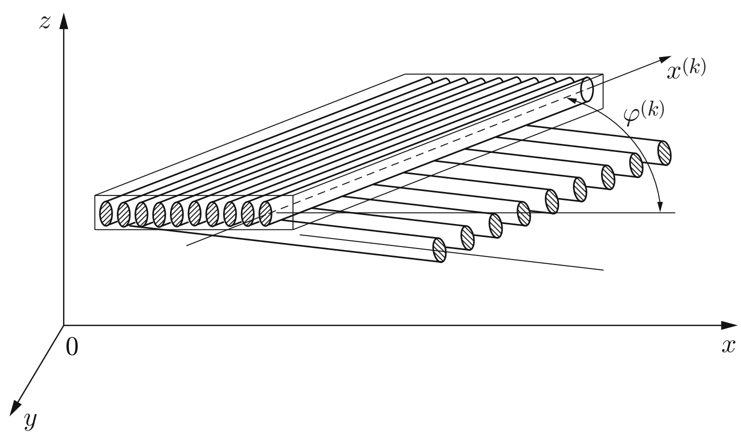

Fig. 5. Orientation of the panel layers.

bending theory with the use of Lekhnitskii's complex potentials [16] defined in the form of Cauchy-type integrals with an unknown function of integrand density over the contours L bounding the plate. With the use of Sokhotsky–Plemelj formulas, the boundary-value problem for analytical functions describing the stress–strain state of the plate,

$$a(t)\Phi_1^+(t) + b(t)\overline{\Phi_1^+(t)} + \Phi_2^+(t) = F(t),$$

is reduced to a singular integral equation with respect to unknown functions of the integrand density

$$\int\limits_L \frac{\omega(\tau)}{\tau_1 - t_1}\,\frac{d\tau_1}{ds}\,ds + \int\limits_L [K_1(t,\tau)\omega(\tau) + K_2(t,\tau)\overline{\omega(\tau)}\,]\,ds = f(t), \qquad t \in L$$

with additional conditions of single-valuedness of displacements in the plate:

$$\int\limits_L \omega(\tau)\,d\tau_1 = 0, \qquad \int\limits_L \omega(\tau)\Big(\tau_1 - \frac{\alpha}{\gamma}\,\tau_2 - \frac{\overline{\beta}}{\overline{\gamma}}\,\overline{\tau_2}\Big)\,d\tau_1 = 0.$$

Singular equations are solved by the method of quadratures based on using an interpolation polynomial for approximating the functions of the integrand density and Gauss–Chebyshev quadrature formulas. Solving the

TABLE 3

Optimal Thicknesses of the Panel Weakened by an Elliptical Hole versus the Number of Points T on the Hole Contour ($a/b = 1/2$)

T	h_{opt}, mm	φ, deg (h, %)			
10	8.460	0 (36)	−45 (3)	90 (29)	45 (32)
30	8.900	0 (29)	−45 (5)	45 (5)	90 (61)
40	8.920	0 (30)	−45 (5)	45 (5)	90 (60)
180	8.897	0 (32)	45 (3)	−45 (3)	90 (62)

TABLE 4

Optimal Thicknesses of the Panel Weakened by an Elliptical Hole versus the Ratio of the Semi-Axes in the Case of Bending by the Moments M_x and H_{xy}

N^p, 10^{-4} N	h_{opt}, mm	φ, deg (h, %)			
		$a/b = 1/2$			
(1; 0; 0)	11.1	−45 (8)	45 (10)	0 (36)	90 (46)
(1; 0; 0.5)	12.3	45 (23)	−45 (34)	0 (43)	90 (0)
(1; 0; 1)	13.2	45 (22)	−45 (27)	0 (25)	90 (26)
(1; 0; 1.5)	14.8	45 (21)	−45 (28)	0 (25)	90 (26)
(1; 0; 2)	16.4	45 (20)	−45 (29)	0 (25)	90 (26)
		$a/b = 1$			
(1; 0; 0)	8.89	0 (30)	−45 (4)	45 (5)	90 (61)
(1; 0; 0.5)	10.8	0 (18)	90 (24)	45 (27)	−45 (31)
(1; 0; 1)	12.1	45 (26)	−45 (26)	90 (24)	0 (24)
(1; 0; 1.5)	14.1	45 (23)	−45 (26)	90 (25)	0 (26)
(1; 0; 2)	16.0	45 (19)	90 (4)	0 (3)	−45 (74)
		$a/b = 2$			
(1; 0; 0)	7.4	0 (34)	90 (22)	45 (21)	−45 (23)
(1; 0; 0.5)	9.7	45 (20)	−45 (9)	90 (8)	0 (63)
(1; 0; 1)	11.3	45 (25)	−45 (25)	90 (25)	0 (25)
(1; 0; 1.5)	13.8	45 (22)	−45 (27)	90 (26)	0 (25)
(1; 0; 2)	15.6	90 (5)	0 (4)	45 (21)	−45 (70)

system of singular equations is reduced to solving a system of algebraic equations with respect to unknown values of the functions of the integrand density at collocation points.

As an example, we describe the process of solving the problem of bending of an unbounded anisotropic multilayer plate weakened by an elliptical hole and loaded by bending and rotational moments at infinity ($M_x^\infty; M_y^\infty; H_{xy}^\infty$). The stress–strain state in the plate can be obtained by summing the stress–strain state in the plate without the hole and disturbances of the stress–strain state induced by the hole [17].

We have to find the minimum weight of the structure by varying the thicknesses and the order of the layers.

Tables 3 and 4 show the results of optimization for a panel consisting of seven layers made of the HMS/3002M material (its elastic and strength characteristics are given in Sec. 3) with the packing angles $\varphi_1 = 0°$, $\varphi_2 = 90°$, $\varphi_3 = -45°$, and $\varphi_4 = 45°$ (the order of the layers can be changed) and weakened by an elliptical hole with semi-axes a and b.

Table 3 gives the optimal thickness, the order of the layers, and the percent ratio of the thicknesses of the panel layers to the total optimal thickness of the panel h (h_1, h_2, h_3, and h_4) bent by the moment $M_x = 10^4$ N versus the number of collocation points on the hole contour T. The numbers of the layers correspond to the order of their location from the outer edge toward the mid-plane. It follows from Table 3 that an accurate approximation of the exact solution can be obtained by using 40 points on the hole contour.

Table 4 shows the results calculated for different values of a/b for a panel loaded by the moments $M_x = 10^4$ N and $M_y = 0$ and for different values of the rotational moment H_{xy}.

Table 5 shows the results calculated by the coordinatewise descent method on a finite interval and by the gradient descent method [17]. The calculations were performed for two variants of the external action and

TABLE 5

Optimal Thicknesses of the Panel Weakened by an Elliptical Hole
Obtained by the Coordinatewise Descent Method and by the Gradient Descent Method

b/a	Gradient descent method					Coordinatewise descent method				
	h_{opt}, mm	φ, deg (h, %)				h_{opt}, mm	φ, deg (h, %)			
$M_y = 10^4$ N and $M_x = H_{xy} = 0$										
0.2	15.3	45 (6)	90 (39)	0 (19)	−45 (36)	15.46	−45 (6)	45 (12)	90 (31)	0 (51)
0.6	10.6	−45 (6)	90 (29)	45 (17)	0 (48)	10.26	45 (7)	−45 (8)	90 (31)	0 (54)
1.0	8.89	90 (31)	−45 (3)	45 (4)	0 (62)	8.68	90 (33)	−45 (21)	45 (1)	0 (45)
1.6	7.79	90 (38)	−45 (2)	0 (60)	—	7.57	90 (39)	−45 (23)	45 (1)	0 (37)
2.0	7.30	90 (35)	45 (3)	0 (28)	−45 (34)	7.10	90 (42)	−45 (26)	0 (32)	—
$H_{xy} = 10^4$ N and $M_x = M_y = 0$										
0.2	9.85	45 (16)	−45 (84)	—	—	9.72	45 (17)	−45 (83)	—	—
0.6	10.5	−45 (16)	45 (84)	—	—	10.42	45 (16)	−45 (84)	—	—
1.0	10.6	45 (16)	−45 (84)	—	—	10.54	45 (16)	−45 (60)	90 (3)	0 (21)
1.6	10.5	−45 (16)	45 (84)	—	—	10.46	45 (16)	−45 (84)	—	—
2.0	10.4	45 (16)	−45 (84)	—	—	10.34	45 (16)	−45 (84)	—	—

Note. The dash means degeneration of the layer.

for different values of b/a. The optimal thickness values are fairly close to each other, but there are significant differences in the ratio of the layer thicknesses for some types of loading and some ratios of the ellipse semi-axes. These differences are usually observed for internal (less loaded) layers, while the thicknesses and orientation of external (load-bearing) layers are in good agreement.

Conclusions. The methods, algorithms, and corresponding computer codes described in the paper allow one to calculate the stress–strain state, to estimate the strength, and to obtain the optimal designs of articles made of composite materials. The problems of weight optimization for a cantilever beam under bending, for a flat panel loaded in its plane, and for a panel weakened by an elliptical hole and loaded by bending moments are solved. The proposed method of optimization is demonstrated to be fairly effective, especially in combination with the method of boundary integral equations, for solving problems of optimal design of composite structures. Owing to their fast operation and universality, the optimization procedures can be used for solving a wide class of problems for various structures made of composite materials.

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